Test Scales and norms

1. *Types of Test Score in Common Use*

The most frequent methods of reporting test results appear to be:

Percentiles
T Scores
I.Q's
Sten Scores

This chapter will describe the nature of each of these types of scale, and give the method of converting scores of one type into scores of another. At the present time it is possible to have data for an individual on a number of tests each of which gives its results differently. It is therefore necessary to be able to compare one type of scale with another.

2. *Percentiles*

Percentiles may be taken as points on a measuring scale below which a stated percentage of cases fall. Thus below the 75th percentile will fall 75 percent of cases, below the 10th percentile 10 percent, and so on. The 50th percentile is the median, and in the case of a normal distribution it is the mean and mode as well. In constructing percentile norms the first step is to tabulate the frequency distribution of the scores. When this has been done a cumulative frequency distribution is worked out. This gives the cumulative total of cases at each score level working usually from the lowest score upwards. Once this has been obtained the percentiles corresponding to the scores are worked out by the formula:

$$\text{Percentile} = \left(\frac{cfB = .5f}{N} \right) \times 100 \tag{4:1}$$

where:

*cf*B = cumulutive frequency of the score below the one for which the percentile is being calculated;

f = frequency of the score whose percentile is being calculated;

N = total number of cases.

The complete procedure is illustrated below in Table 4:1.

TABLE 4:1 OBTAINING PERCENTILE NORMS FROM FREQUENCY DISTRIBUTIONS

Score	*Frequency of Score(f)*	*Cumulative Frequency of Score (cf)*	*cfB + .5f*	*Percentile (cfB + .5f)/N* × 100
50	2	20	19	95
49	3	18	16.5	82.5
48	4	15	13	65
47	5	11	8.5	42.5
46	3	6	4.5	22.5
45	2	3	2	10
44	1	1	.5	2.5

It should be emphasized, if it is not immediately apparent, that percentile norms would never be worked out on such a small number of cases. The process can also be worked in the opposite direction. If we want to find the score corresponding to a given percentile we use the formula:

Score corresponding to a percentile

$$= XLL + W\left(\frac{Np - cfB}{f}\right) \qquad (4:2)$$

where:

XLL = the lower limit of the class interval, in this case the scores are in units, so *XLL* = X -.5.
e.g. *XLL* for *X*50 = 49.5, for *X*37 = 36.5 etc.

W = Width of class interval in units.
e.g suppose scores were classed 90-94, 95-99, 100-104. *W* would be 5. In Table 4:1 *W* = 1.

Np = Number of cases times the proportion indicated by the percentile. Referring this value to the cumulative frequency column enables us to find the interval in which the value corresponding to the percentile will lie.

cf B and *f* are as above in formula 4:1.

As an example suppose with the data in Table 4:1 we wanted to find the 75th percentile, we would take the following steps:

Step 1. The value of *Np* will be 20 x .75 = 15. So we need the interval containing the 15th case.

Step 2. This corresponds to a score of 48, and in this interval are 4 cases, so *f* = 4.

Step 3. *cfB* = 11.

Step 4. The lower limit of the interval containing the 75th percentile is 47.5.

Step 5. Putting these together we get:

$$47.5 + 1\left(\frac{15 - 11}{4}\right) = 48.5 \qquad = \left\lfloor XLL + W\left(\frac{Np - cfB}{f}\right)\right\rfloor$$

3. *T Scores*

T scores are normally distributed scores with a mean of 50 and a standard deviation of 10. If the distribution of obtained scores is normal then *T* scores can be worked out directly by:

(a) converting each score to a *Z* score;

(b) multiplying the *Z* by 10; then

(c) adding or subtracting (depending on the sign of the *Z*) to or from 50. This will be the *T* score.

$$T = 10\left(\frac{X - M}{\sigma_x}\right) + 50 \qquad (4:3)$$

If the scores are not normally distributed the *T* scores will have to be calculated from percentiles. This procedure incidentally has the effect of normalizing the distribution of scores. Table 4:2 contains the raw scores and percentiles from Table 4:1. Two columns have been added. One of these gives the *Z* values corresponding to percentiles. These were obtained from tables for the normal curve, and the other gives *T* score values.

TABLE 4:2 CALUCLATION OF *T* SCORES FROM RAW SCORES

Score	*Percentile*	*Z Score*	*T Score (50 + 10Z)*
50	95	+1.64	66
49	82.5	+0.93	59
48	65	+0.39	54
47	42.5	-0.19	48
46	22.5	-0.76	42
45	10	-1.28	37
44	2.5	-1.96	30

It will be seen that differences of one unit in the original scores are represented on the *T* scale as varying between 5 and 7.

Problems

A. (a) What T score will correspond to the 5th percentile,
(b) the 16th percentile,
(c) the 99th percentile,
(d) the 50th percentile?

B. What is the score corresponding to the median in Table 4:1?

Answers

A. (a) 34, (b) 40, (c) 73, (d) 50.
B. 47.3.

4. *IQ*'s

IQ's are intelligence quotients and are used in reporting intelligence test scores. Nowadays nearly all I.Q's are deviation I.Q's. Raw scores are converted to a scale with a given mean and standard deviation. In the case of Wechsler Scales the mean is 100 and the standard deviation is 15. If raw scores are normally distributed one can convert them into I.Q's with a mean of 100 and standard deviation of 15 by:

(1) Converting to *Z* scores.
(2) Multiplying the *Z* by 15.
(3) Adding or subtracting the result from 100.

A formula which does this is:

$$I.Q.(M,100;\sigma,15) = \frac{(X-M)}{\sigma} + 100 \qquad (4:4)$$

To convert I.Q's into percentiles it is necessary to convert them to *Z* scores and then find the percentile by reference to tables for the normal curve.

Problems

A. Given raw scores with a mean of 80 and a standard deviation of 20, convert the following into I.Q's on a scale with a mean of 100 and a standard deviation of 15.

(a) 20; (b) 90; (c) 100; (d) 67.

A. Now convert each one to an I.Q. on a scale with a mean of 100 and a standard deviation of 24.

B. Convert the following I.Q's to percentiles. The test has a mean of 100 and a standard deviation of 15.

(a) 100; (b) 90; (c) 130; (d) 115;
(e) 110; (f) 70; (g) 85.

Answers

A. (a) 55; (b) 107.5 (c) 115; (d) 90.

B. (a) 28; (b) 112 (c) 124; (d) 84.

C. (a) 50th; (b) 25th; (c) 98th; (d) 84th; (e) 75th; (f) 2nd; (g) 16th.

5. *Sten Scores*

The main tests on which sten scores are used are the personality questionnaires prepared by R. B. Cattell and his associates. Cattell defines stens as:

'Units in a standard ten scale, in which ten score points are used to cover the population range in fixed and equal standard deviation intervals, extending from $2^1/_2$ standard deviations above the mean (sten 10). The mean is fixed at 5.5 stens.' (Cattell, 1965, *The Scientific Study of Personality,* London: Pelican, p.374.

The units on the sten scale are thus half a standard deviation in width, which is fairly coarse grouping compared with *T* scores, (one tenth of a standard deviation), and Wechsler I.Q's, one fifteenth of a standard deviation. Each sten covers a range of percentiles as shown in Table 4:3 which also shows the range of *T* Scores and Wechsler type I.Q's corresponding to stens. This last information is included because Factor B of the 16 Personality Factor Questionnaire is a measure of intelligence.

TABLE 4:3 THE RELATIONSHIP BETWEEN STEN SCORES AND OTHER SCORES

	Upper limit of sten	*Percentiles at upper limits*	*T Scores of range covered by the Sten*	*I.Q's*
10				
9	9.5	97.72	70	130
8	8.5	93.32	65	122.5
7	7.5	84.13	60	115
6	6.5	69.15	55	107.5
5	5.5	50	50	100
4	4.5	30.85	45	92.5
3	3.5	15.85	40	85
2	2.5	6.68	35	77.5
1	1.5	2.28	30	70

(No upper limit is given for Sten 10, and no lower limit for Sten 1, as these would not be sensible.)

Problems

An individual obtains the following test results on a series of intelligence tests.

(a)	Test A (*M* 100, σ 15)		145
(b)	Test B		84th percentile
(c)	Test C	*T* Score	61
(d)	Test D (*M* 100, σ 24)		148
(e)	Test E	Sten Score	9

Convert these scores into:

A. Sten Scores
B. Percentiles.
C. I.Q's (*M* 100, σ 15).
D. *T* Scores.
E. Scores on a test with (*M* 500, σ 50).

Answers

A. (a) 10; (b) 7; (c) 8; (d) 10.
B. (a) 99.9; (c) 86.4; (d) 98; (e) 93-98 percentile.
C. (b) 115; (c) 117; (d) 130; (e) 122-130.
D. (a) 80; (b) 60; (d) 70; (e) 65-70.
E. (a) 650; (b) 550; (c) 555; (d) 600; (e) 575-600.

6. *A Table showing the Relationships between Percentiles, Z Scores, Wechsler I.Q's and T Scores*

TABLE 4:4 THE RELATIONSHIPS BETWEEN PERCENTILES, *Z* SCORES, WECHSLER I.Q'S AND *T* SCORES

Percentile	*Z Score*	*I.Q.*	*T Score*
1st	- 2.33	65	27
5th	- 1.64	75	34
10th	- 1.28	81	37
15th	- 1.04	84	40
20th	- 0.84	87	42
25th	- 0.67	90	43
30th	- 0.52	92	45
35th	- 0.39	94	46
40th	- 0.25	96	48
45th	- 0.13	98	49
50th	0.00	100	50
55th	+ 0.13	102	51
60th	+ 0.25	104	52
65th	+ 0.39	106	54
70th	+ 0.52	108	55
75th	+ 0.67	110	57
80th	+ 0.84	113	58
85th	+ 1.04	116	60
90th	+ 1.28	119	63
95th	+ 1.64	125	66
99th	+ 2.33	135	73

7. *A general Formula for converting Scores on a Scale with given Mean and Standard Deviation into Scores on a Scale with different Mean and Standard Deviation*

In the previous sections of this chapter the conversion of scores from one scale to another has usually been through the use of *Z* scores. This has been done to emphasize the logic of the procedure. The steps have been:

(1) find the *Z* score on Scale 1

$$\left(\frac{X_1 - M_1}{\sigma_1} = Z\right)$$

and

(2) convert the *Z* score to a score on Scale 2 by (a) multiplying the *Z* score by the standard deviation of
Scale 2 and (b) adding the mean

$$\left(X_2 = Z\sigma_2 + M_2\right)$$

But as the *Z* score on both Scales will be the same by definition:

$$X_2 = \left(\frac{\sigma_2}{\sigma_1}\right)X_1 - \left[\left(\frac{\sigma_2}{\sigma_1}\right)M_1 - M_2\right] \qquad (4:5)$$

Proof

(1) $\dfrac{X_2 - M_2}{\sigma_2} = \dfrac{X_1 - M_1}{\sigma_1}$, by definition.

(2) Multiplying both sides by σ_2 gives:

$$X_2 - M_2 = \left(\frac{\sigma_2}{\sigma_1}\right)(X_1 - M_1) = \left(\frac{\sigma_2}{\sigma_1}\right)X_1 - \left(\frac{\sigma_2}{\sigma_1}\right)M_1$$

(3) Adding M_2 to both sides gives

$$X_2 = \left(\frac{\sigma_2}{\sigma_1}\right)X_1 - \left(\frac{\sigma_2}{\sigma_1}\right)M_1 + M_2$$

(4) Thus:

$$X_2 + \left(\frac{\sigma_2}{\sigma_1}\right)X_1 - \left[\left(\frac{\sigma_2}{\sigma_1}\right)M_1 - M_2\right]$$

For many purposes this formula is easier to use than the procedure using *Z* scores. As an example of its use suppose that it is desired to convert a WAIS I.Q. of 90 into a *T* Score. Substituting the appropriate value in the formula gives:

$$T\text{ Score } = \left(\frac{10}{15}\right)90 - \left[\left(\frac{10}{15}\right)100 - 50\right] = 43.$$

In any situation where there are a large number of scores to convert from one scale to another, the fact that:

$$\frac{\sigma_2}{\sigma_1}(M_1 - M_2)$$

will be a constant will ease the computational burden. If only one score is to be converted, the following variant of (4:5) should be used

$$X_2 = \frac{\sigma_2}{\sigma_1}(X_1 - M_1) + M_2 \qquad (4:6)$$